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ANALYSIS OF SIZE EFFECT，SHEAR DEFORMATION AND DILATION IN DIRECT SHEAR TEST BASED ON GRADIENT-DEPENDENT PLASTICITY

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Abstract The paper concerns the issue of size law，localized deformation and dilation or compaction due to shear localization. It is assumed that the shear localization initiates at the peak shear stress in the form of single shear band，and based on gradient-dependent plasticity，an analytical solution on size effect or snap-back is obtained. The results show that the post peak response becomes steeper and even exhibits snap-back with increasing of length. For small specimen，the relative shear displacement when specimen failure occurs is lower than that of larger specimen and the shear stress-relative displacement curve becomes steeper. The theoretical solution on non-uniformity of strains in shear band is obtained and evolution of the relative shear displacement is represented. By resorting to the linear relation between local plastic shear strain and local plastic volumetric strain，the dilation and compaction within shear band are analyzed. Relation between apparent shear strain and apparent normal strain and relation between shear displacement and vertical displacement are established.

Key words gradient-dependent plasticity，direct shear test，shear localization，size effect，dilation，compaction，shear band

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1 INTRODUCTION

The direct shear test is a widely used soil or rock test that provides essential design data in stability analysis of slope，foundation and rockburst and so on. The test is inevitably subject to criticisms because of the non-uniformity of stress and strains，which may facilitate the occurrence of progressive failure along the potential shear plane[1]. However，the test is the standard method for the measurement of plane strain strength parameters. To obtain a full understanding of the test，some experimental observations[1～3] and numerical experimental using the finite element method[4～5] or distinct element method[6～7] have been adopted.

However，it should be pointed out that little effort has been made in the past to study the test through analytical solution，owing to that the analytical solution can not deal with non-uniform plastic shear strain in shear band based on classical plastic theory in which stress only depends on strain.

One of the most promising approaches proposed to model localization relies on the incorporation of

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higher gradients， typically second order spatial gradients in the yield condition.

In the present paper，gradient-dependent plasticity is used to analyze the size effect，snap-back，shear deformation and dilation in direct shear test. Compared with earlier experimental or numerical results， the validity of the analytical solution is checked.

2　ANALYSIS

The constitutive relation between shear stress and shear strain is bi-linear. In elastic range，the shear stress τ is computed by the shear modulus G via the linear relation：$\tau = G\gamma$. In the plastic range we consider linear strain softening in terms of the non-local plastic shear strain as soon as $\tau_{\rm c}$ is attained. The slope of curve $\tau-\gamma$ in strain softening region is $-\lambda$ and λ is called shear descending modulus and its value is always positive. The onset of strain localization is at the peak shear stress in the form of single shear band parallel to the direction of shear stress τ shown in Fig.1. After localization initiates，the width of the localized band is w. The rock with total length L outside the shear band remains intact and is elastically unloaded. For simplicity，the shear band is treated as a one-dimensional shearing problem. In the center of the band，$y = 0$.

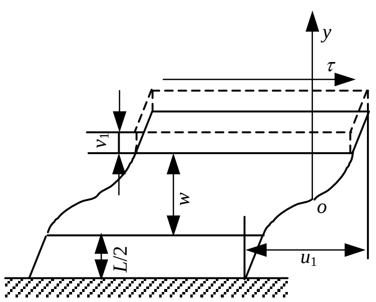

Fig.1　Definitions for the direct shear test

According to Ref.[8，9]，local plastic shear strain in shear band can be expressed as

$$\gamma^{\rm p} = \frac{\tau_{\rm c}-\tau}{c}\left(1+\cos\frac{y}{l}\right) \tag{1}$$

where $c = \dfrac{G\lambda}{G+\lambda}$，$c$ is called softening modulus. Total local plastic shear strain in shear band is computed by

$$\gamma = \frac{\tau}{G} + \frac{\tau_{\rm c}-\tau}{c}\left(1+\cos\frac{y}{l}\right) \tag{2}$$

When $|y| \leqslant \dfrac{w}{2}$，we can obtain the relative shear displacement across the shear band：

$$u_1 = 2\int_0^y \gamma\,{\rm d}y = \frac{2\tau}{G}y + \frac{2(\tau_{\rm c}-\tau)}{c}\left(y + l\sin\frac{y}{l}\right) \tag{3}$$

Similarly，when $|y| \geqslant \dfrac{w}{2}$，the relative shear displacement is

$$u_1 = 2\int_0^y \gamma^{\rm e}{\rm d}y + 2\int_0^{\frac{w}{2}} \gamma^{\rm p}{\rm d}y = \frac{2\tau}{G}y + \frac{(\tau_{\rm c}-\tau)w}{c} \tag{4}$$

For the case of $y = \dfrac{w}{2}+\dfrac{L}{2}$，we can obtain the relative shear displacement between upper top and the lower surface of the rock specimen：

$$u_1 = \frac{\tau}{G}(w+L) + \frac{(\tau_{\rm c}-\tau)w}{c} \tag{5}$$

Differentiation of Eq.(5) yields

$$\frac{{\rm d}\tau}{{\rm d}u_1} = \frac{G\lambda}{L\lambda - wG} \tag{6}$$

The equation above represents the size effect and the behavior of snap-back. It is assumed that in elastic region，the volumetric strain of rock is zero. After strain localization initiates，the dilatancy is caused by local plastic shear strain $\gamma^{\rm p}$ in terms of Ref.[10～14]

$$\sin\psi = -\frac{\varepsilon_{\rm v}^{\rm p}}{\gamma^{\rm p}} \tag{7}$$

where $\varepsilon_{\rm v}^{\rm p}$ is local plastic volume strain，ψ is dilation angle. The local plastic volumetric strain can be determined by local increment of volume/original volume ratio：

$$\varepsilon_{\rm v}^{\rm p} = -\frac{{\rm d}v_y}{b{\rm d}y} \tag{8}$$

where b is shear area，${\rm d}v_y$ is local increment of volume. Substitution of Eq.(1) and (7) in Eq.(8) results in

$${\rm d}v_y = b\sin\psi\frac{\tau_{\rm c}-\tau}{c}\left(1+\cos\frac{y}{l}\right){\rm d}y \tag{9}$$

The increment of volume ΔV within shear band can be computed by

$$\Delta V = \int {\rm d}v_y = 2b\sin\psi\frac{\tau_{\rm c}-\tau}{c}\int_0^{\frac{w}{2}}\left(1+\cos\frac{y}{l}\right){\rm d}y =$$

$$bw\sin\psi\frac{\tau_{\rm c}-\tau}{c} \tag{10}$$

Supposing that the shear dilation occurs only in the direction of y axis，we can get

$$\Delta V = bv_1 \tag{11}$$

where v_1 is the vertical displacement due to shear dilation. Using Eq.(10) and (11)，we can obtain

$$v_1 = w\sin\psi\frac{\tau_c-\tau}{c} = w\sin\psi(\tau_c-\tau)\left(\frac{1}{\lambda}+\frac{1}{G}\right) \tag{12}$$

3　EXAMPLES

3.1　Shear deformation

It is well-known that non-uniform strains exist in direct shear test，e.g. under fully softened condition a large concentration of plastic strains in the potential shear plane which crosses horizontally the entire sample can be observed in numerical results[4]. Adopting distinct element method，Masson et al.[7] performed an analysis of the behavior of a granular material. For dense sample，due to high material shearing strain at the split plane level，particle velocity vectors are quite erratic. Nevertheless，the instantaneous view of the particle velocity field exhibits marked shear localization at the interface between the two parts of the box. Using the same method，Liu et al.[6] obtained distribution of mean horizontal displacements along specimen height in direct box shear test. These results show that in the plastic region，shear band whose thickness is dependent on particle diameter is localized within a thin layer of material and in the shear band the shear strain field is strongly heterogeneous.

To investigate the non-uniformity of strains in shear band，we take parameters as follows：$w = 0.013$ m，$G = 2$ GPa，$\lambda = 0.2\,G$，$L = 0.001$ m and $\tau_c = 2$ MPa. The relative horizontal shear displacement of different flow shear stress in direct shear test is depicted in Fig.2. The theoretical solution not only reproduces the non-uniformity of strains in shear band，but also represents the evolution of the relative shear displacement. The lower the flow shear stress is，the larger the shear displacement is. For lower flow shear stress，the displacement has larger value and higher shear strain gradient exists within shear band once the fully softened condition is attained. However，outside the shear band，the shear strain is uniform and its value decreases with non-local plastic shear strain.

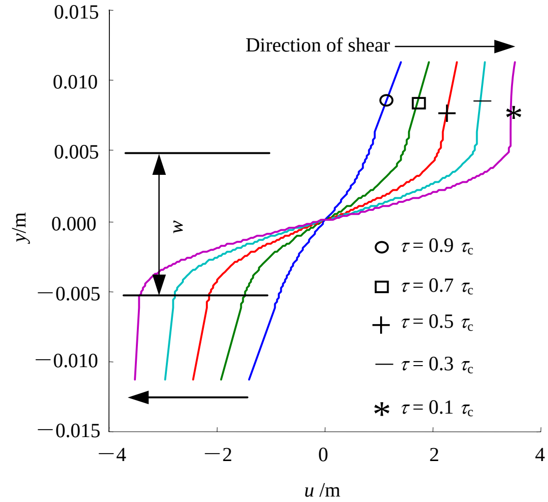

Fig.2　Horizontal displacement of direct shear test

3.2　Size effect

According to Bazent[15]，the problem of size effect is particularly important to geotechnicas researcher who must inevitably extrapolate from reduce-scale laboratory tests to real structures. Compared with size effect in direct shear test，the size effect of compression，tension and three-point bend beam has attracted more attentions. In the authors' knowledge，the analytical solution on size effect in direct shear test has not been put forward so far. By using the large size single shear apparatus，the single shear tests for fine grained soil-concrete interface are conducted by Gao et al.[16]. The size effect can be observed manifestly in the experimental results. (1) For small soil specimen，the relative displacement measured when failure occurs is lower than that of large specimen. (2) However，the ratio of relative displacement to length ratio of specimen (i.e. $L+w$) ratio increases with decreasing size of the specimen. (3) The experiments also shows that the shear stress-relative displacement curve becomes steeper with decreasing size of the specimen.

In order to study the influence of size on response of direct shear test，we take parameters as follows：$G = 20$ GPa，$w = 0.004$ m，$\lambda = 0.1\,G$ and $\tau_c = 20$ MPa. Fig.3 presents the relation between shear stress and relative displacement. Notice that the relative displacement is large for large specimen when shear strength is attained，as is consistent with experimental results (1). Moreover，in elastic region，the slope of shear stress-relative displacement curve tends to increase as the size of specimen decreases，which is in agreement with above experimental results (3). The validity of the experimental results (2) will be discussed subsequently.

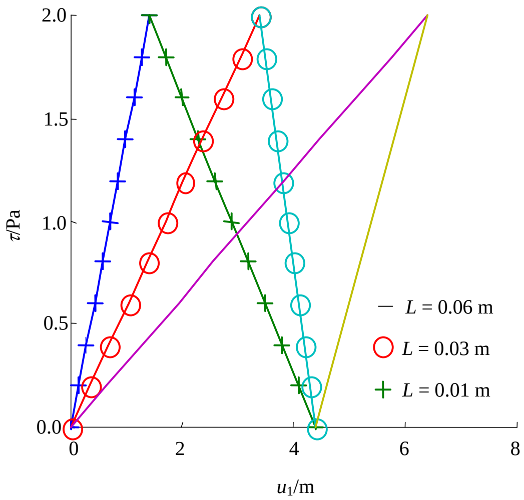

Fig.3　Size effect of direct shear test

It is noted that the post peak response becomes steeper and even exhibits snap-back with increasing length in present model for direct shear test. For quasi-brittle materials，the similar phenomenon or so called Class II behavior can be often observed in many kinds of experiments，such as uniaxial compression or tension，torsion and three-point bend，and theoretical analysis[17，18].

3.3　Dilation and compaction

From Eq.(12)，we can see that the vertical displacement due to shear dilation is proportional to the width of the shear band，the sine of dilation angle，the differential shear stress and the value of c. That is to say，larger particle diameter or dilation angle can lead to more remarkable dilation. Under fully softened condition，the dilation attains its maximum. However，at the peak stress，the dilation of volume is zero. For more brittle rock material，lower dilation can be expected because the cracks or fissures can not propagate easily in entire shear band.

It is well known that the dense sample dilates in direct shear tests，while the loose sample compacts. The analysis above can be also applied to loose sample if $\psi<0$.

From Eq.(5) and Eq.(12)，we can obtain a linear relation between horizontal and vertical displacement

$$u_1=\frac{\tau}{G}(w+L)+\frac{v_1}{\sin\psi} \tag{13}$$

It should be noted that the term $\tau(w+L)/G$ is elastic horizontal displacement，while the term $v_1/\sin\psi$ is plastic horizontal displacement caused by localized deformation. The relation between u_1 and v_1 is shown in Fig.4.

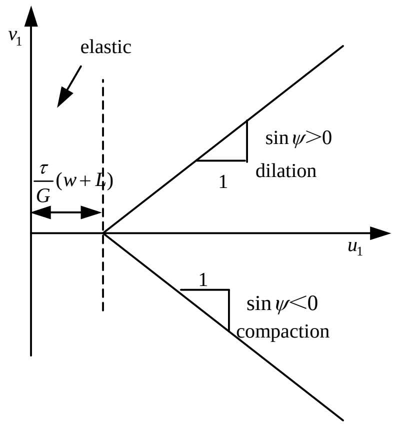

Fig.4　Relation between horizontal and vertical displacements

Using Eq.(13)，we can get the ratio of relative displacement to length of specimen ratio

$$\frac{u_1}{w+L}=\frac{\tau}{G}+\frac{v_1}{\sin\psi(w+L)} \tag{14}$$

Seen from Eq.(12)，v_1 is not dependent on the length of the elastic zone. For a kind of material，the constitutive parameters，such as G，ψ，w and λ are all constants. On the condition of specimen failure，the flow shear stress τ is also a specific value. So，for a small specimen corresponding to low height of the elastic zone L，the radio of relative displacement to length of specimen ratio is large. Herein，the rationality of the experimental results (2) mentioned above is interpreted.

In fact，the ratio of relative displacement to length ratio of specimen ratio can be referred to as apparent or mean shear strain，while the ratio of vertical displacement to length of specimen ratio can be called apparent or mean normal strain. So，we can establish the relation between apparent shear strain and apparent normal strain shown in Fig.5，i.e.

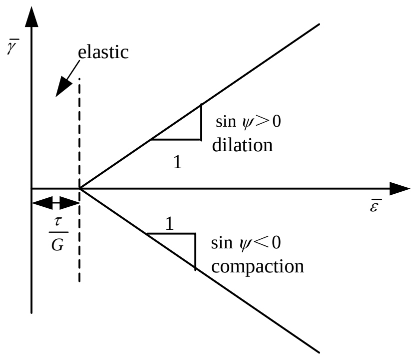

Fig.5　Relation between $\bar{\gamma}$ and $\bar{\varepsilon}$

$$\bar{\gamma}=\frac{\tau}{G}+\frac{\bar{\varepsilon}}{\sin\psi} \tag{15}$$

It is noted that the two parameters $\bar{\gamma}$ and $\bar{\varepsilon}$ can be measured in experiments. If the elastic part of $\bar{\gamma}$ is neglected，we can get a simplified formula，i.e. apparent shear strain is proportional to apparent normal strain. Similarly，through neglecting the elastic displacement，Eq.(13) can also be simplified. The simplified theoretical results can be observed in some experimental and numerical results[1～7].

4　CONCLUSIONS

Based on gradient-dependent plasticity，the non-uniformity of strains and the evolution of the relative shear displacement are represented. The post peak response in direct shear test becomes steeper and even exhibits snap-back with increasing length. The vertical displacement due to shear dilation is proportional to the width of the shear band，the sine of dilation angle，the differential shear stress and the value of softening modulus. Plastic apparent shear strain is proportional to apparent normal strain. Moreover，plastic shear displacement is also in proportion to vertical displacement

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